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Reg. No. : .....

Name : .....



Fourth Semester B.Sc. Degree Examination, July 2023

First Degree Programme under CBCSS

Mathematics

Complementary Course for Physics

MM 1431.1 — MATHEMATICS IV – FOURIER SERIES, COMPLEX ANALYSIS AND PROBABILITY THEORY

(2021 Admission)

Time : 3 Hours Max. Marks : 80

PART – A

Answer **all** questions. Each question carries **1** mark.

1. State the Cauchy's Integral formula.

2. Find the real part of $f(z) = z^2 + 3z$.

3. Find the Fourier sine series of the function $f(x) = x$, $0 \le x < \pi$.

4. Find the residue of $e^{\frac{1}{z}}$ at $z = 0$.

5. A pack of 52 cards two are drawn at random. Find the probability that both are aces.

6. State Baye's theorem.

P.T.O.

7. State Cauchy – Riemann equation.

8. If $F(s)$ is the Fourier transform of $f(x)$ then what is the Fourier transform of $f(x-a)$ where a is any real number.

9. Given an example for an even function.

10. Out of 50 members of a club in how many ways can a president, vice president and a secretary can be chosen.

(10 × 1 = 10 Marks)

PART – B

Answer any **eight** questions. Each question carries **2** marks.

11. Find all the values of $(-i)^{\frac{1}{3}}$.

12. Define periodic function and give an example.

13. Distinguish between discrete and continuous random variables.

14. State Laurent's theorem.

15. Find the probability that a single card drawn from a shuffled deck of cards will be either a diamond or king.

16. Evaluate $\oint_C \frac{z+1}{z^2} dz$, where C is the unit circle.

17. Expand $f(x) = (x-1)^2$, $0 < x < 1$ in a Fourier series of sine terms only.

18. Determine residue of $f(z) = \frac{\cos z}{z^4}$, at $z = 0$.

19. Find the Fourier sine transform of $f(x) = e^{-ax}$, where $a > 0$.

20. A random variable X has the density function $f(x) = kx$ for $2 \le x \le 5$. Find k.

21. Find the Binomial distribution whose mean is 9 and variance is 6.

22. Find the half range cosine series of $f(x) = x$, $0 < x < \pi$.

(8 × 2 = 16 Marks)

PART – C

Answer any **six** questions. Each question carries **4** marks.

23. Expand $\frac{1}{z^2-3z+2}$ in Laurent's series valid in the region $1<|z|<2$.

24. Find the mean and variance of a random variable X which takes the values 0, 1, 2, 3 with respective probabilities $\frac{1}{8}, \frac{3}{8}, \frac{3}{8}, \frac{1}{8}$.

25. Find the Fourier series of the function $f(x)=|x|$, $-\pi \le x \le \pi$.

26. Evaluate $\int_C \tan z\, dz$ where C is $|z|=2$.

27. Find the Fourier Transform of e^{-x^2}.

28. Prove that $U = x^3 - 3xy^2 - 5y$ is harmonic and find its harmonic conjugate.

29. Find the Fourier sine transform of $\frac{e^{-ax}}{x}$.

30. If the probability of a bad reaction from a certain injection is 0.001, determine the chance that out of 2000 individuals more than two will get a bad reaction.

31. A problem in mathematics is given to three students A, B and C whose chances of solving it are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$ respectively. What is the probability that the problem will be solved?

(6 × 4 = 24 Marks)

PART – D

Answer any **two** questions. Each question carries **15** marks.

32. (a) There are 3 bags: first containing 1 white, 2 red, 3 green balls; second 2 white, 3 red, 1 green balls and third 3 white. 1 red, 2 green balls. Two balls are drawn from a bag chosen at random. These are found to be one white and one red. Find the probability that the balls so drawn came from the second bag.

(b) In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and S.D of the distribution.

33. Find the Fourier series of the function $f(x)=\begin{cases}-\pi & \text{if } -\pi < x < 0 \\ x & \text{if } 0 < x < \pi\end{cases}$ and Compute the sum $1+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+.....$

34. (a) Use Cauchy's integral formula to evaluate $\int_C \frac{z+1}{z^4+2iz^3}dz$ where $C\ |z|=1$.

(b) Evaluate $\int_C \bar{z}\,dz$ where C is given by $x=3t$, $y=t^2$, $1\le t\le 4$.

35. (a) Evaluate $\int_0^{2\pi} \frac{\cos 3\theta}{5-4\cos\theta}d\theta$.

(b) Suppose two dice are thrown. If X denote the sum of the numbers on the dice, find the probability density function of X.

(2 × 15 = 30 Marks)
